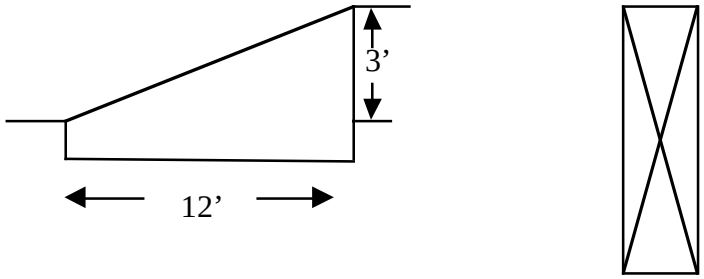
Twelfth Annual DOE-LLNL Math Challenge - May 12, 2001

1. I am building a sloped ramp across a channel that is 12 feet wide, and which has one side 3 feet higher than the other, as shown. The ramp is a rectangular wooden board 4 feet wide and just long enough to span the channel. I will reinforce the ramp with wooden beams spanning each of the diagonals of the ramp, as shown, slotted at the center where they cross. To the nearest .1 foot, how long, in feet, is each of these diagonal beams?

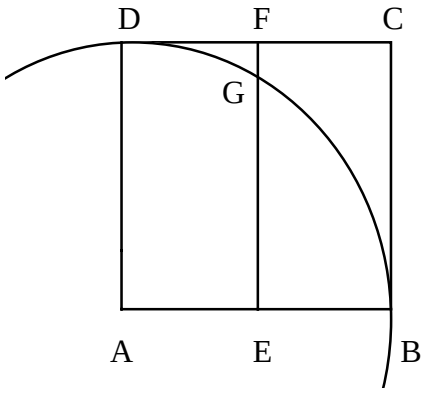


2. If $f(x) = \dfrac{x^2 - 15x + 36}{x^3 - 14x^2 + 21x + 36}$, for x such that the denominator is not zero, find f(f(f(f(x)))).

3. Simplify the expression $\sin^2 x\,(1 - \cot^4 x + \csc^4 x)$ to just one term.

4. If persons A and B choose an integer at random from 1 to 300 independently, what is the probability that B chooses a larger number than A?

5. Using the approximations, $\log_{10}(2) = .3010$ and $\log_{10}(7) = .8451$, how many consecutive zeros following the decimal point are there in $(.98)^{1000}$?



6. In the square ABCD of side 1 shown above, the midpoints of AB and CD are E and F, respectively. The circle of radius 1 centered at A meets EF at point G. What is the size of the angle DCG?

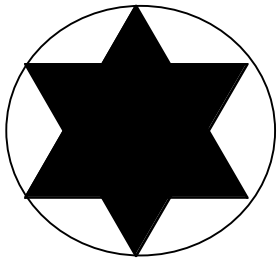
7. How many roots does the equation, $\sin x = \sin 2x$, have in the interval $-\pi < x < \pi$?

8. A garden contains red, yellow, and blue flowers, with all three colors represented. A visitor observes that any bunch of three flowers you pick will contain a red flower. Another visitor observes that any bunch of three flowers you pick will contain a yellow flower. How many flowers of each color are in the garden?

9. In Sherwood Forest, there are deer, foxes, and rabbits, with a total of 42 such animals in all. Among the foxes and rabbits, the numbers of males and females are equal in each species, but for the deer the females outnumber the males by a ratio of 4:3. Among the female animals, there are as many rabbits as there are deer and foxes combined. There are as many male deer as there are foxes of both sexes. How many male deer are there?

10. Find all two-digit numbers such that, when a 0 is inserted between their two digits, the numbers are increased by 5 hundreds minus 5 tens.

11. Find the area of the regular six-pointed star inscribed in a circle of radius 1 shown below.



12. How many leap years are there in the current millennium, that is, years 2001 through 3000? A year is a leap year if it is divisible by 4, except if it is divisible by 100, but the exception does not apply if it is divisible by 400.

13. A rocket travels at 4/5 the speed of light from the earth to the star Alpha Centauri, 4 light years away. A clock on the rocket measures time in years starting from takeoff. Because of relativity, the clock on the rocket is slowed relative to earth time by a factor of $\sqrt{1 - v^2/c^2}$, where v is the speed of the rocket and c is the speed of light. If started at zero, what time, in years, will the rocket's clock show when it arrives at Alpha Centauri?

14. Two "dice" in the shape of regular icosahedra, each with faces numbered 1, 2, ...,20, are rolled on a table. The number on the face that ends up on the table surface is read from each die, and the two numbers are added. What is the most likely resulting number, and what is the probability of getting it?